# An Extension of the Omega Method to Primary and Tertiary Creep of Lead-Free Solders

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**Abstract**

A close examination of creep curves for Sn-based solders shows that steady-state creep is an ill-defined stage and that primary and tertiary creep are both significant. In order to account for these effects in solder constitutive models, this paper presents a phenomenological model that predicts entire creep curves for Sn-based solders. The proposed constitutive model is an extension of the Omega method (Prager, 2000) for tertiary creep of steel alloys to primary and tertiary creep of Sn-based solders. The new creep modeling approach is of use for stress/strain and reliability analysis of lead-free solder joints.

## Application of Classical Creep Models to Sn-Based Solders

Classical solder creep models give total creep strains as the sum of primary and secondary or steady-state creep strains:

$$\varepsilon_{CREEP} = \varepsilon_{SAT}(1-\exp(-K\overset{\circ}{\varepsilon}_{SS}\cdot t))+\overset{\circ}{\varepsilon}_{SS}\cdot t \qquad (1)$$

The parameters in equation (1) are: $\varepsilon_{SAT}$, the primary saturation strain; K, a rate constant for exponential decay of primary strains as in the Pao-Marin (1957) primary creep model; and $\overset{\circ}{\varepsilon}_{SS}$, the steady-state or minimum strain rate for a given stress and temperature. From (1), instantaneous creep rates are:

$$\overset{\circ}{\varepsilon}_{CREEP} = \varepsilon_{SAT}\cdot K\overset{\circ}{\varepsilon}_{SS}\cdot\exp(-K\overset{\circ}{\varepsilon}_{SS}\cdot t)+\overset{\circ}{\varepsilon}_{SS} \qquad (2)$$

Tertiary creep strains are not considered in solder constitutive models, in general. Indeed, modeling of solder tertiary creep has received little attention, with perhaps one exception, the application of the modified Theta projection concept to Sn-Ag solder (Kariya et al., 2003).

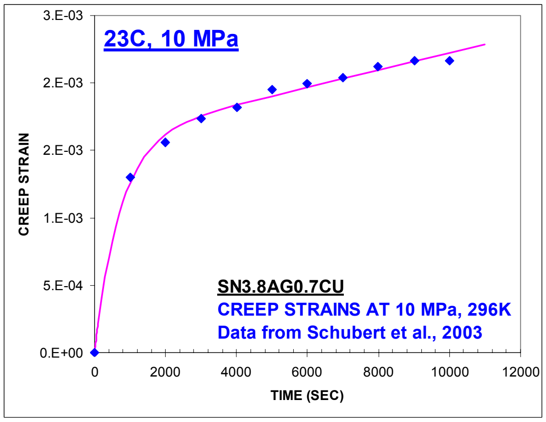
Figures 1-4 show four examples of curve-fitting of equations (1) and (2) to creep strain data and the corresponding creep rates for Sn3.5Ag and SAC solders under constant stress and temperature. A cursory look at the data and the fitted curves gives rough estimates of the duration of primary and secondary creep stages (Table 1).

| ALLOY | TEMPERATURE, STRESS | PRIMARY CREEP | SECONDARY CREEP |
|---|---|---|---|
| SAC 3807 | 23°C, 10 MPa | ~ 100 MINUTES | NA |
| | 110°C, 10 MPa | ~ 17 MINUTES | ~ 50 MINUTES |
| SN3.5AG | 75°C, 19.9 MPa | ~ 7 MINUTES | ~ 100 MINUTES |
| | 125°C, 20 MPa | ~ 10 SECONDS | ~ 1 MINUTE |

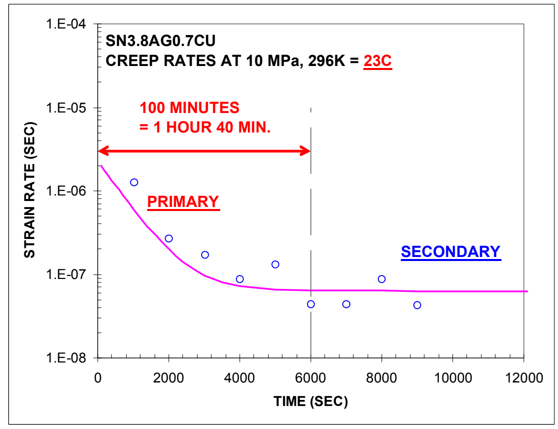
**Table 1:** Estimated duration of primary and secondary creep stages for creep curves shown in Figures 1-4.

Depending on stress and temperature for the creep curves shown in Figures 1-4, primary creep lasts from 10 seconds to 100 minutes, and steady-state appears to last from 1 minute to 100 minutes. The duration of the primary creep stage can be significant in comparison to time-scales encountered in common accelerated thermal cycling profiles. The secondary creep stage is of limited duration and constitutive models that do not account for tertiary creep will underestimate creep strains. This is shown as the departure of the model lines from the data in the tertiary region of creep curves in Figures 2a, 3a and 4a. The duration estimates in Table 1 are user-dependent and subjective since there is no unique criterion to define when primary creep ends or when steady-state creep starts and ends. The above observations are in agreement with measurements by Plumbridge (2003) who showed that, at 75°C, the extent of steady-state creep for various Sn-based solders (SnPb, SnCu, SnAg and SAC) was in the range 17% to 38% of the creep rupture time. Plumbridge's criterion for the duration of steady-state creep was the period of time for which creep rates are within a margin of 10% above the minimum creep rate. Using a similar criterion, Shohji et al. (2004) observed that secondary creep is from 1% to 48% of the creep rupture life for Sn8.0ZnBi3.0 solder at 25°C, 75°C and 100°C and that the extent of steady state decreased, with tertiary creep becoming more dominant, as applied stresses decreased.

The application of classical creep models to constitutive modeling of Sn-based solders suffers from two possible limitations. First, as shown in the examples above, steady-state creep is assumed to last for ever, which underestimates creep strains in the tertiary stage. Second, since time appears as a state variable in equation (2), the formulation of instantaneous creep rates does not satisfy the principle of objectivity of constitutive modeling (e.g., Stouffer et al., 1996). While equation (2) adequately fits creep rate data in the primary and secondary creep regions, its use in a constitutive creep rate model implies that solders respond to a time variable. The principle of objectivity states that "*constitutive equations must be invariant under a change of reference. The principle embodies both spatial and temporal reference changes... The observed response must be invariant under a time and spatial change of reference frame*" (Stouffer et al., 1996). Time is not a recommended variable in creep rate formulations such as equation (2), especially when used in numerical simulations, since the calculated rate would change under a simple translation in time.

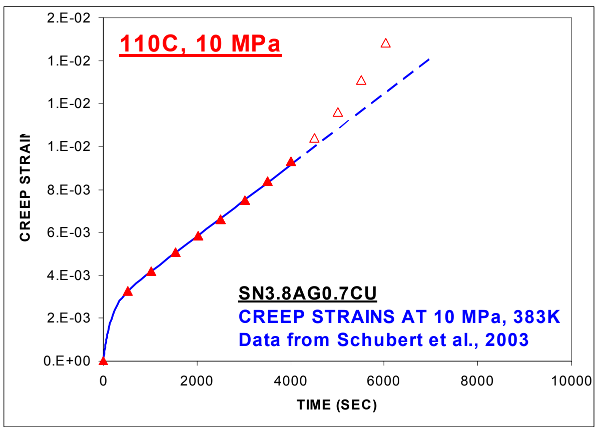



(a) Creep strain versus time and fit of creep model

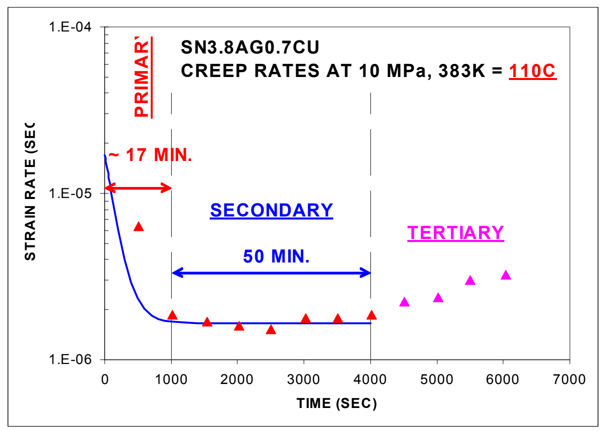



(b) Creep rates versus time and fit of creep rate model

**Figure 1:** Example 1: Fit of classical creep models to SAC creep strain and strain rate data (original creep data at 10 MPa, 23°C after Schubert et al., 2003)

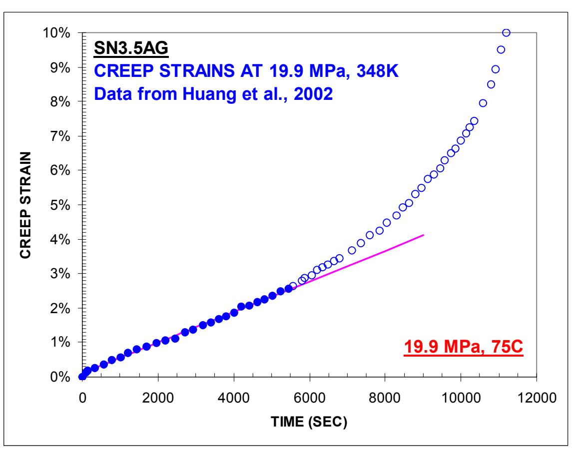



(a) Creep strain versus time and fit of creep model

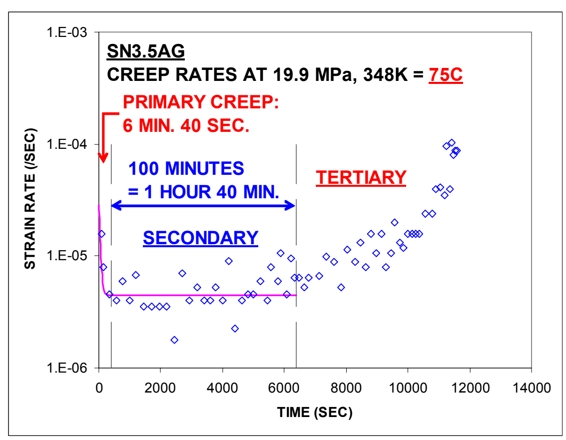



(b) Creep rates versus time and fit of creep rate model

**Figure 2:** Example 2: Fit of classical creep models to SAC creep strain and strain rate data (original creep data at 10 MPa, 110°C after Schubert et al., 2003)

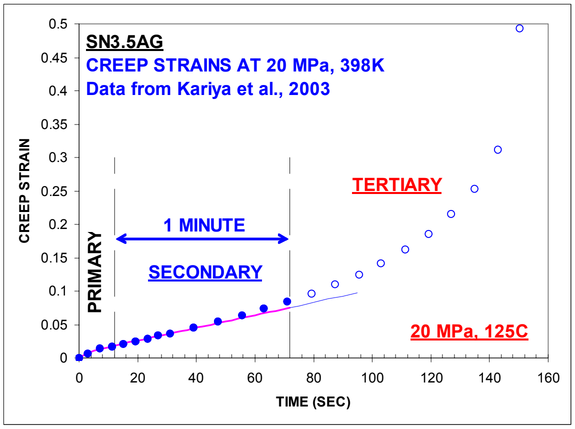



(a) Creep strain versus time and fit of creep model

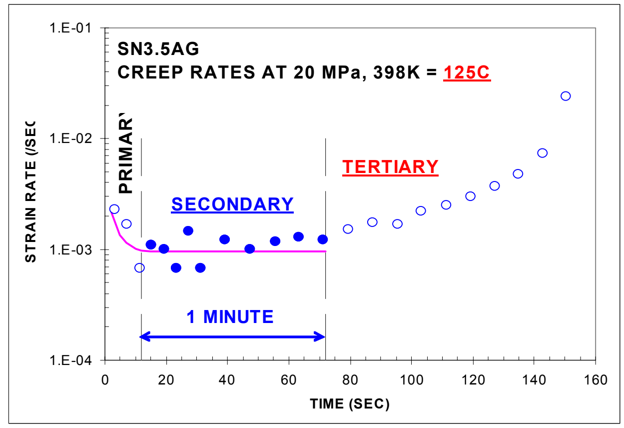



(b) Creep rates versus time and fit of creep rate model

**Figure 3:** Example 3: Fit of classical creep models to Sn3.5Ag creep strain and strain rate data (original creep data at 19.9 MPa, 75°C after Huang et al., 2002)

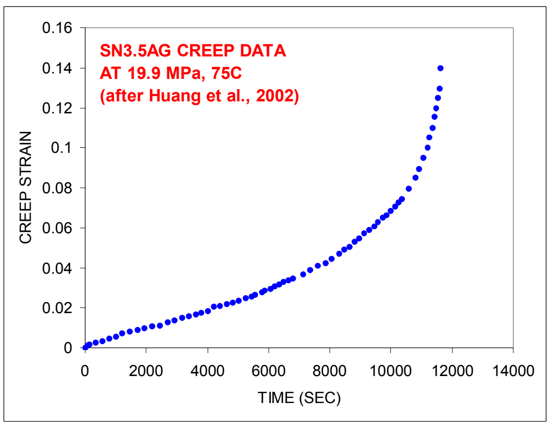



(a) Creep strain versus time and fit of creep model

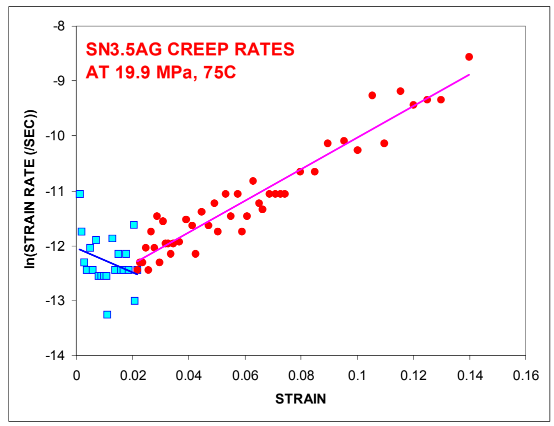



(b) Creep rates versus time and fit of creep rate model

**Figure 4:** Example 4: Fit of classical creep models to Sn3.5Ag creep strain and strain rate data (original creep data at 20 MPa, 125°C after Kariya et al., 2003)

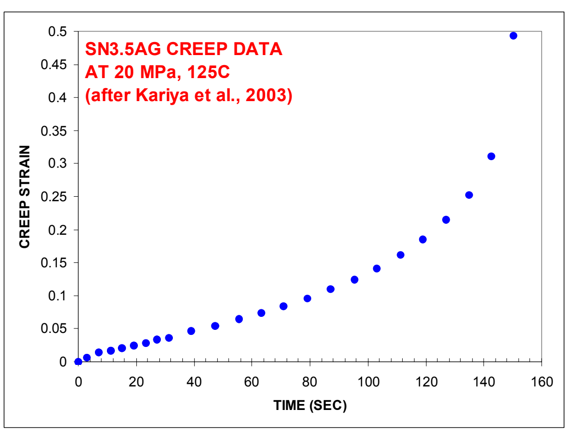



(a) Original creep data

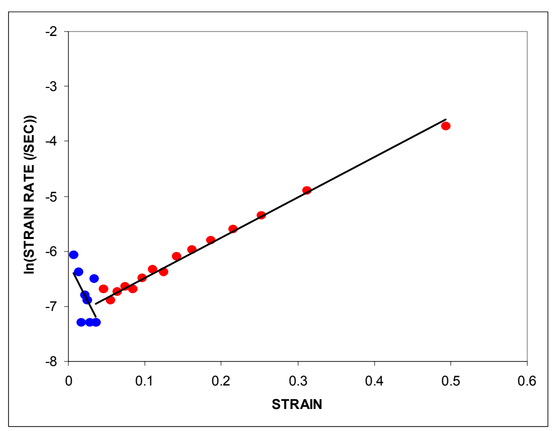



(b) Strain rate vs. strain data (log-linear)

**Figure 5:** Example 5: Sn3.5Ag creep curve and log-linear plot of strain rates vs. strains (original creep data at 19.9 MPa, 75°C after Huang et al., 2002)

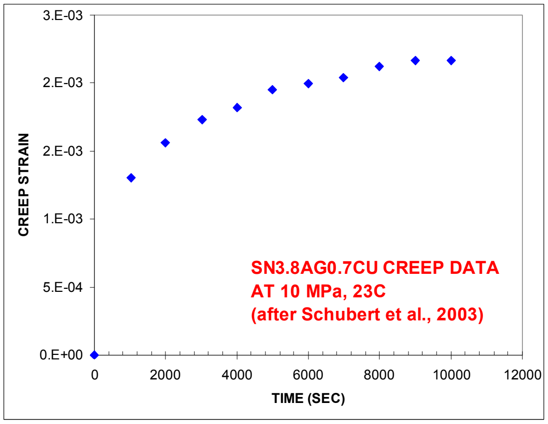



(a) Original creep data

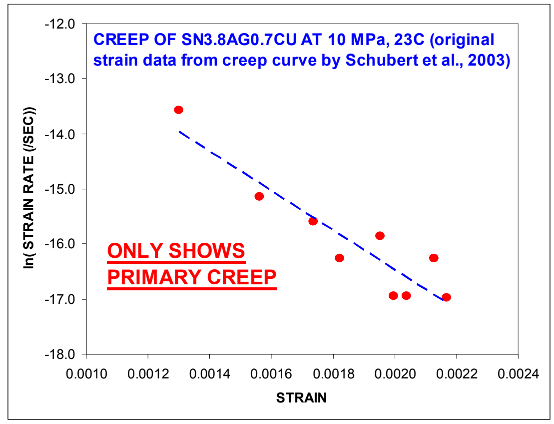



(b) Strain rate vs. strain data (log-linear)

**Figure 6:** Example 6: Sn3.5Ag creep curve and log-linear plot of strain rates vs. strains (original creep data at 20 MPa, 125°C after Kariya et al., 2003)

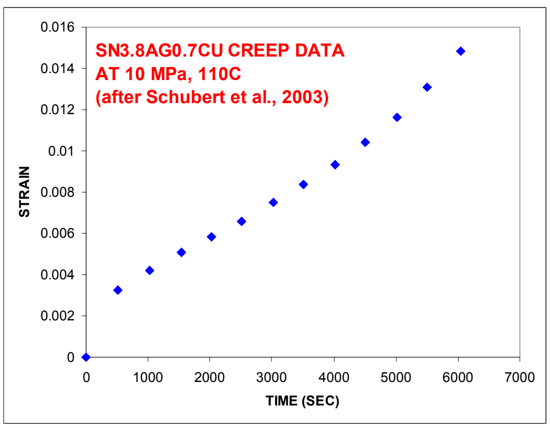



(a) Original creep data

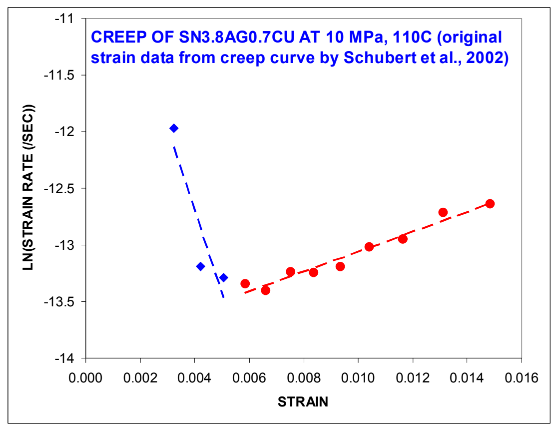



(b) Strain rate vs. strain data (log-linear)

**Figure 7:** Example 7: SAC creep curve and log-linear plot of strain rates vs. strains (original creep data at 20 MPa, 23°C after Schubert et al., 2003)

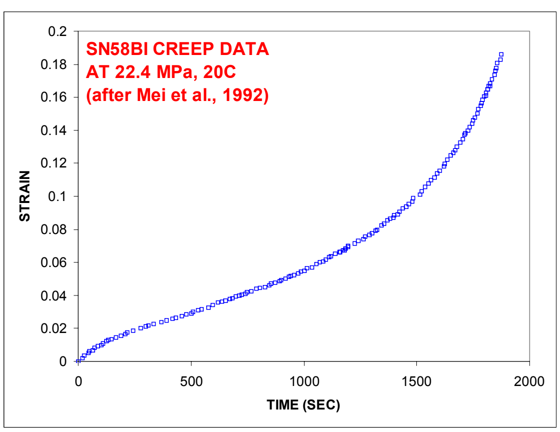



(a) Original creep data

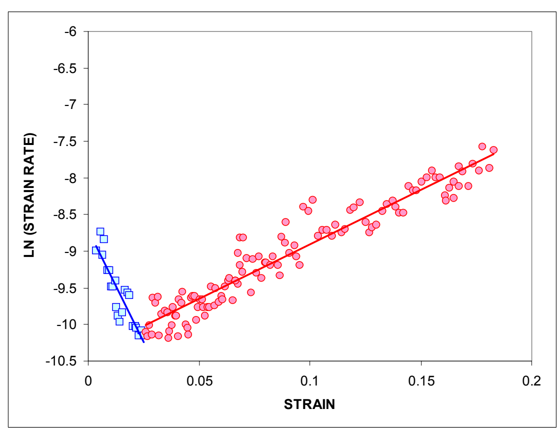



(b) Strain rate vs. strain data (log-linear)

**Figure 8:** Example 8: SAC creep curve and log-linear plot of strain rates vs. strains (original creep data at 10 MPa, 110°C after Schubert et al., 2003)

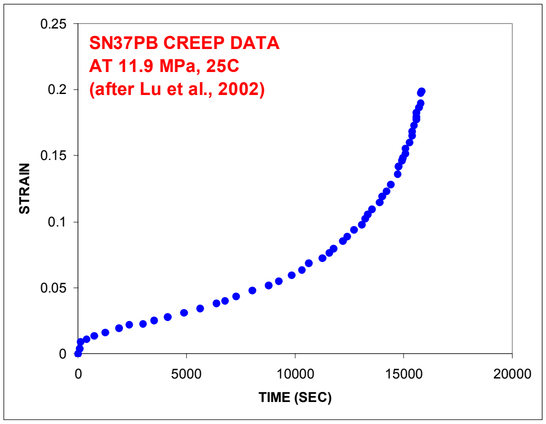



(a) Original creep data

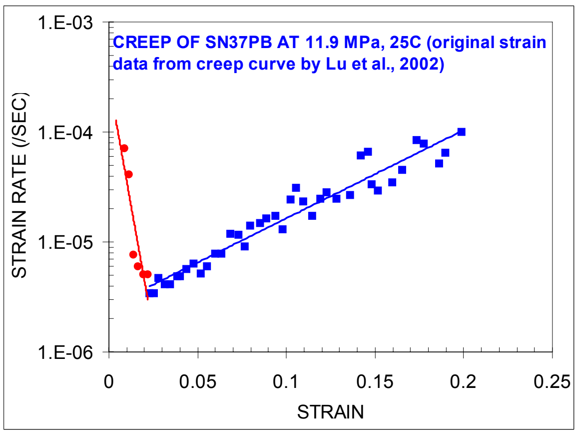



(b) Strain rate vs. strain data (log-linear)

**Figure 9:** Example 9: Sn58Bi creep curve and log-linear plot of strain rates vs. strains (original creep data at 22.4 MPa, 20°C after Mei et al., 1992)




(a) Original creep data




(b) Strain rate vs. strain data (log-linear)

**Figure 10:** Example 10: Sn37Pb creep curve and log-linear plot of strain rates vs. strains (original creep data at 11.9 MPa, 25°C after Lu et al., 2002).

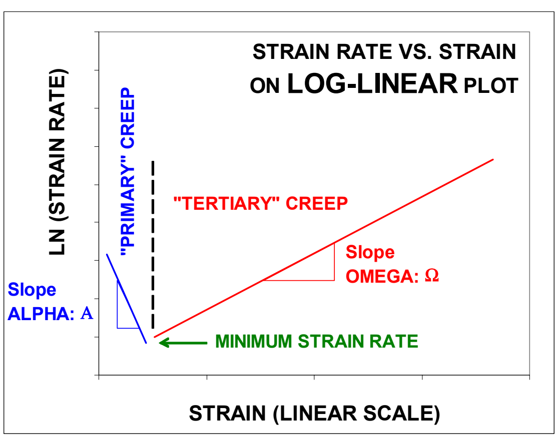



**Figure 11:** Schematic of trends in log-linear plot of strain rate versus strain data.

**Extension of the Omega Model to Sn-Based Solders**

Figures 5-10 show creep curves and the corresponding plots of strain rate versus strain in LOG-LINEAR coordinates for a variety of Sn-based solders. The latter plots show regions of decelerating strains, a minimum creep rate, and accelerating creep strains. The area where the minimum creep rate occurs corresponds to an inflexion point in creep curves but there is no extended or well-defined region where the minimum creep rate holds steady. The strain at the minimum creep rate, not time, determines the transition from dominant hardening in the region of decelerating strains to dominant softening in the region of accelerating strains.

As shown schematically in Figure 11, the log-linear plots of the strain rate vs. strain data follow linear trends with slopes ALPHA (A) in the region of decelerating strains or primary creep, and OMEGA (Ω) in the region of accelerating strains or tertiary creep. This suggests a simple phenomenological model for Sn-based solders where strain rates in the regions of decelerating and accelerating strains follow the respective equations:

$$\ln(\overset{\circ}{\varepsilon}) = -\mathrm{A} \cdot \varepsilon + C_1 \qquad (3a)$$

$$\ln(\overset{\circ}{\varepsilon}) = \Omega \cdot \varepsilon + C_3 \qquad (3b)$$

where the intercepts $C_1$ and $C_3$ are constants for a given creep curve. Equations (3a) and (3b) can be rewritten in the form:

$$\overset{\circ}{\varepsilon} = a \cdot e^{-\mathrm{A} \cdot \varepsilon} \qquad (4a)$$

$$\overset{\circ}{\varepsilon} = b \cdot e^{\Omega \cdot \varepsilon} \qquad (4b)$$

where a and b are coefficients associated with a given creep curve. Equation (4b) is the mathematical formulation of the Omega method (Prager, 2000) for tertiary creep of steel alloys. The combined creep rate equation for solders is written as:

$$\overset{\circ}{\varepsilon} = a \cdot e^{-\mathrm{A} \cdot \varepsilon} + b \cdot e^{\Omega \cdot \varepsilon} \qquad (5)$$

The first term on the right hand side of (5) represents strain hardening in the primary creep region of creep curves and the second term captures strain softening or creep damage accumulation in the tertiary creep region. From (4a) and (4b):

$$\frac{\partial \ln(\overset{\circ}{\varepsilon})}{\partial \varepsilon} = -\mathrm{A} \quad \text{or} \quad \Omega \qquad (6)$$

in the primary and tertiary creep regions, respectively. Thus, the slopes A and Ω represent a constant change in the logarithm of creep rates during the hardening or softening stages. A is a hardening parameter for exponential decay of primary creep rates with increasing strains while Ω is a softening (or weakening) parameter for exponential growth of tertiary creep rates with increasing strains, as in the original Omega method for tertiary creep of steels (Prager, 2000). Equation (5) does not include an explicit time-variable and satisfies the principle of objectivity of recommended constitutive modeling practices (Stouffer et al., 1996). In general, equation (5) does not have a closed form solution; however, it is easily integrated using conventional numerical methods.

As shown later, the coefficients "a" and "b" – which have units of strain rates – have a linear dependence with minimum creep rates, and thus, a strong stress and temperature dependence. The slopes A and Ω – which have no units since they represent the inverse of strains – have a weak stress and temperature dependency. Since equation (5) accounts for both decelerating and accelerating strains, the proposed phenomenological model is a natural extension of the Omega method for tertiary creep of steel alloys (Prager, 2000). For convenience, and to reflect that the slopes A and $\Omega$ are significant parameters of the extended $\Omega$ model for creep of Sn-based solders, the proposed model is labeled the Alpha-Omega (A-$\Omega$) model.

**Model Properties In The Primary Creep Region**

For zero creep strain ($\varepsilon = 0$) at time zero, equation (4a) is integrated in closed form to give primary creep strains as a function of time, t:

$$\frac{1}{A}\left(e^{A\varepsilon} - 1\right) = a \cdot t \tag{7a}$$

or $$\varepsilon = \frac{1}{A}\ln(1 + aA \cdot t) \tag{7b}$$

From (7b), the instantaneous primary creep rate as a function of time is:

$$\overset{\circ}{\varepsilon} = \frac{a}{1 + aA \cdot t} \tag{8}$$

Equation (7b) for primary creep strain as a function of time is quite different from that of the Pao-Marin (1957) kinetic model (first term on right hand side of equation (1)). However, at small strains, equation (4a) can be approximated as:

$$\overset{\circ}{\varepsilon} \approx a \cdot (1 - A \cdot \varepsilon) \tag{9}$$

Assuming zero creep strain at time zero, the closed form integration of (9) gives:

$$\varepsilon = \frac{1}{A}\left[1 - e^{-(A \cdot a)t}\right] \tag{10}$$

and the A-$\Omega$ model reduces to the Pao-Marin (1957) model in the region of small primary creep strains. From (10), the slope A of the A–$\Omega$ model is the inverse of the primary creep saturation strain – $\varepsilon_{SAT}$ in equation (1).

**Model Properties In The Tertiary Creep Region**

The reader is referred to Prager (2000) for detailed properties and benefits of the Omega method for tertiary creep of steels. Note also that the Omega method has been adopted as a standard of the American Petroleum Institute (API Recommended Practice # 579). In the tertiary creep region, equation (5) reduces to (4b) which makes the constant "b" the initial creep rate at the beginning of the tertiary creep stage. Thus: $b = \overset{\circ}{\varepsilon}_{MIN}$, the actual minimum creep rate. Equation (4b) is integrated in closed form (Prager, 2000) between time t and the rupture time $t_r$:

$$e^{-\Omega \cdot \varepsilon} - e^{-\Omega \cdot \varepsilon_f} = \Omega b \cdot (t_r - t) \tag{11}$$

where $\varepsilon_f$ is the creep ductility (or rupture strain) at time $t_r$. When $t << t_r$ and the corresponding strain $\varepsilon$ is much smaller ($\varepsilon \rightarrow 0$) than the rupture strain $\varepsilon_f$, which itself is generally large, equation (11) simplifies to:

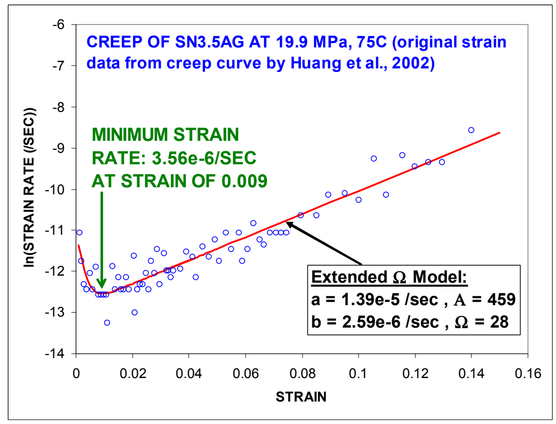
$$1 - 0 \approx \Omega b \cdot t_r \tag{12}$$

i.e.: $$t_r \cdot \overset{\circ}{\varepsilon}_{MIN} \approx 1/\Omega \tag{13}$$

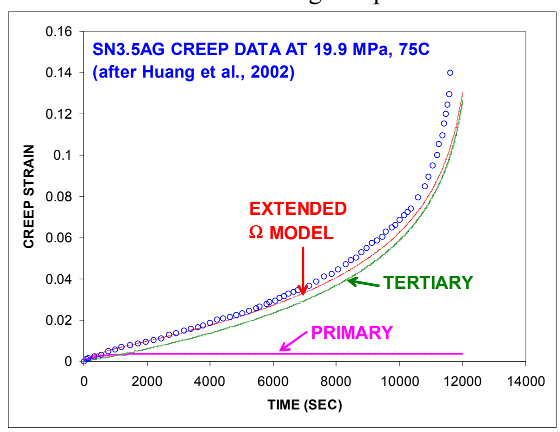
Thus, as in the limiting case of the Omega method (Prager, 2000), the tertiary part of the model reduces to a Monkman-Grant (1956) type of relationship for creep rupture times:

$$t_r \cdot (\overset{\circ}{\varepsilon}_{MIN})^m = C \tag{14}$$

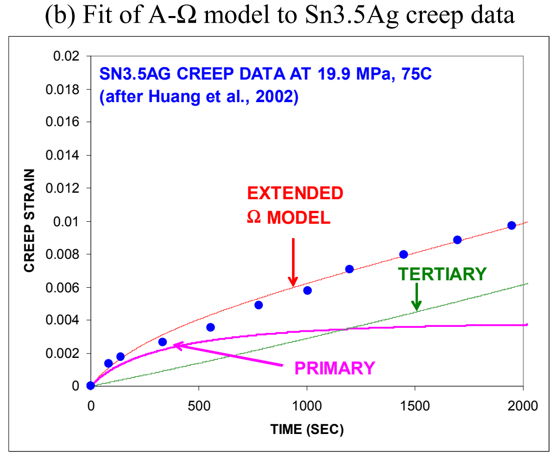
where the exponent m is close to 1 and the constant C, which is related to creep ductility (Plumbridge, 2003), has a weak temperature and stress dependency for soft solders.




(a) Fit of A-$\Omega$ model to Sn3.5Ag creep rate and strain data

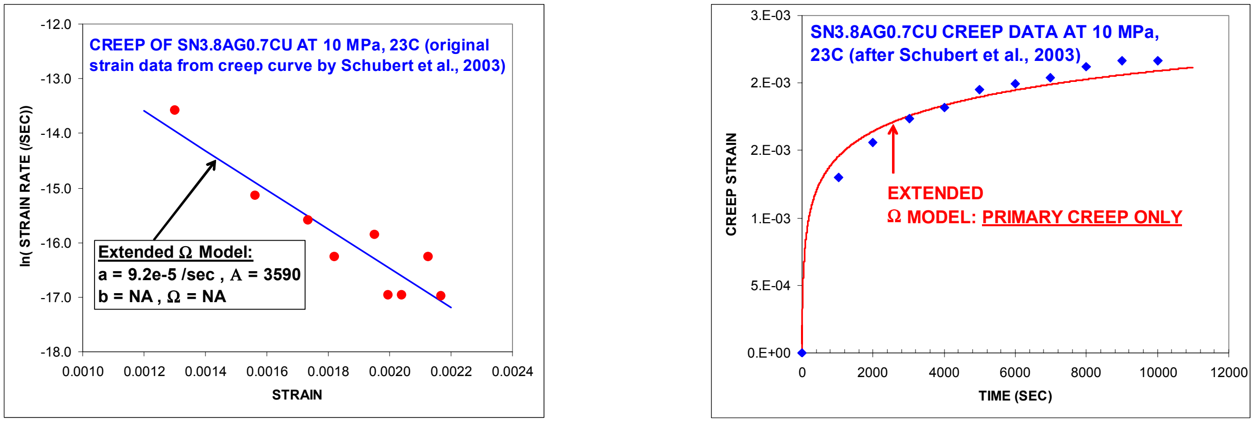



(b) Fit of A-$\Omega$ model to Sn3.5Ag creep data




(c) Magnified view of initial part of curves in (b)

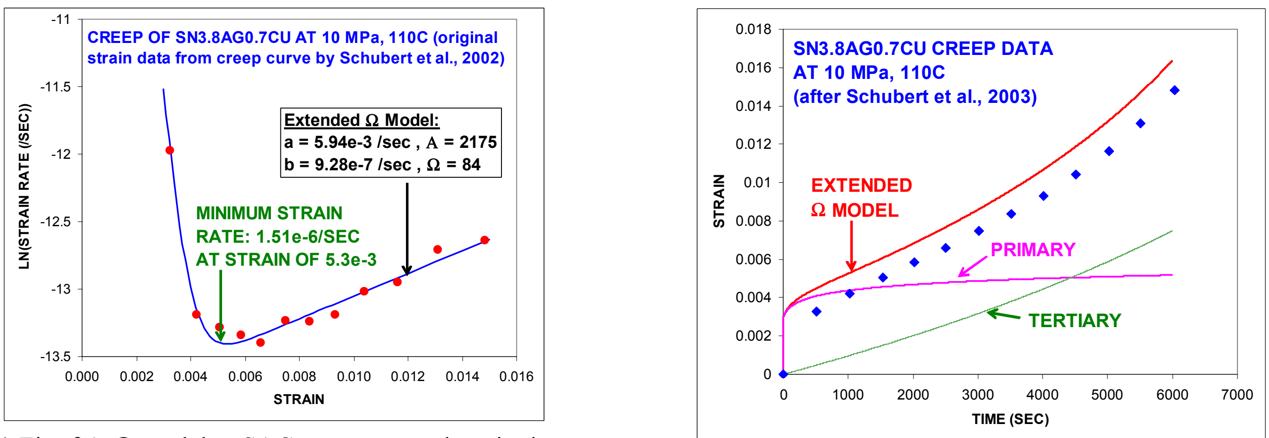
**Figure 12:** Fit of A-$\Omega$ model to Sn3.5Ag creep rate and creep data (original data after Huang et al., 2002).




(a) Fit of A-Ω model to SAC creep rate and strain data

(b) Fit of A-Ω model to SAC creep data

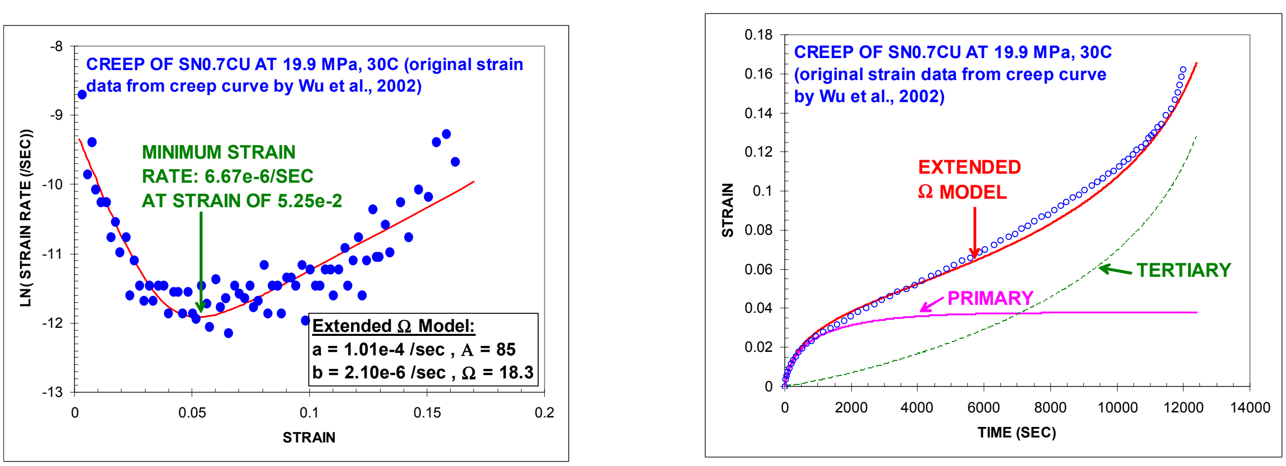
**Figure 13:** Fit of A-Ω model to SAC creep rate and creep data at 23°C (original data after Schubert et al., 2003)




(a) Fit of A-Ω model to SAC creep rate and strain data

(b) Fit of A-Ω model to SAC creep data

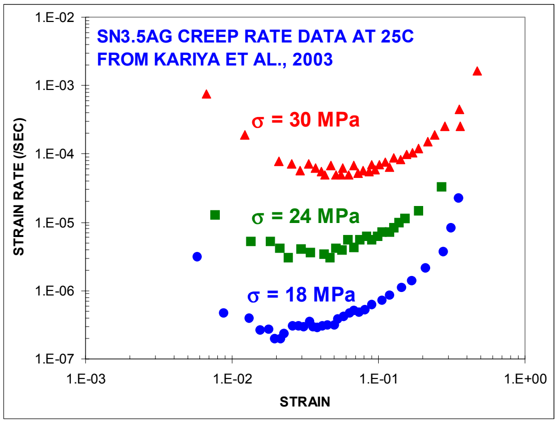
**Figure 14:** Fit of A-Ω model to SAC creep rate and creep data at 110°C (original data after Schubert et al., 2003)



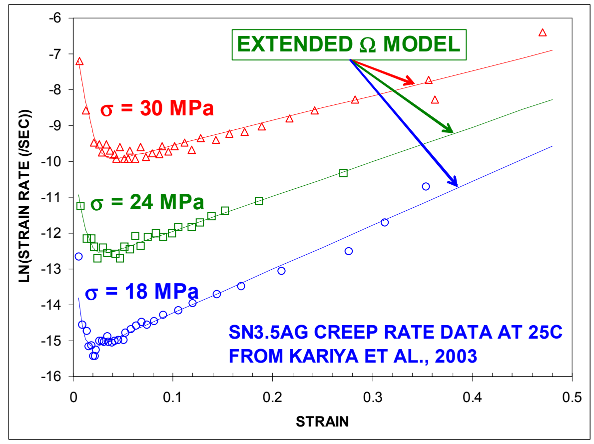

(a) Fit of A-Ω model to Sn0.7Cu creep rate and strain data

(b) Fit of A-Ω model to Sn0.7Cu creep data

**Figure 15:** Fit of A-Ω model to Sn0.7Cu creep rate and creep data at 30°C (original data after Wu et al., 2002)

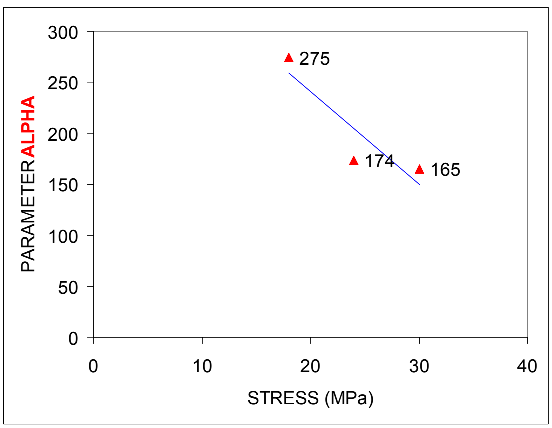



(a) Original data in log-log coordinates

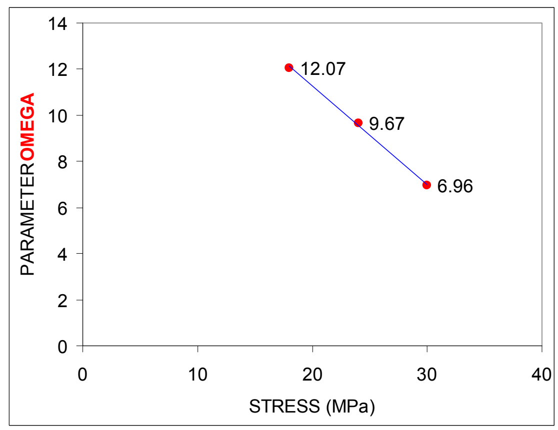



(b) Strain rate vs. strain data in log-linear coordinates

**Figure 16:** Fit of A-Ω model to Sn3.5Ag creep rate and creep data at 25°C and stresses of 18, 24 and 30 MPa (original data after Kariya et al., 2003)




(a) Alpha vs. stress




(b) Omega vs. stress

**Figure 17:** Stress dependence of parameters A and Ω for Sn3.5Ag at 25°C.

The similarity between (13) and (14) implies that we can expect a similarly weak dependency for Ω. Equation (13) itself implies that Ω is a creep damage parameter (Prager, 2000) and that the minimum creep rate has an effect on the entire creep curve.

**Application Example # 1: Sn3.5Ag Creep Data**

The extended Ω model is fitted to Sn3.5Ag creep data as shown in Figure 12. Initial estimates of A and Ω are obtained by linear curve-fitting of the ln(strain rate) versus strain data in the primary and tertiary creep regions (Figure 12a). Actual values of parameters are obtained by non-linear curve fitting of the combined rate model - equation (5) - to the entire dataset of ln(strain rate) versus strain data. Once the parameters and coefficients of (5) are available, the primary and tertiary creep curves are obtained by numerical integration of equations (4a) and (4b). The total creep strain as a function of time is calculated by integration of equation (5) or by adding up primary and tertiary creep strains. The resulting creep curve and its contributions from primary and tertiary creep are shown in Figure 12b (magnified in Figure 12c). In this example, primary creep dominates for a couple of minutes and tertiary creep picks up after about 20 minutes (~ 1200 seconds). From the values of Ω and the minimum creep rate in Figure 12a, equation (13) gives: $t_r \cdot (3.56\times10^{-6}) = 1/28 \Rightarrow t_r = 10{,}032$. The predicted rupture time is 14% less than the experimental $t_r$ ~ 11607 seconds estimated from the actual creep curve. As seen in Figure 12b, the tertiary stage lasts for a relatively long period of time. The corresponding part of the log-linear representation of the data in Figure 12a is linear over a large strain range. If one is only interested in characterizing creep deformations, not creep rupture, the slope Ω can be obtained rapidly (from the initial data in the tertiary region of Figure 12a) after the minimum strain rate has been reached and the test does not have to be carried to rupture. Moreover, once an initial estimate of Ω is available, equation (13) allows for a quick estimate of the creep rupture time. This ability to reduce test time, without carrying the test to rupture, is similar to benefits of the original Omega method (Prager, 2000).

**Application Example # 2: SAC Creep Data**

For SAC solder at 23°C, 10MPa (Figure 13), the log-linear plot of strain rate vs. strain data (Figure 13a) only shows a region of decelerating strains. Thus, only the primary part of the model – equation (4a) – is fit to the experimental data, with parameters as given in Figure 13a. Figure 13b shows the fit of the model to the original creep measurements. SAC solder is still in the primary creep region after over 10,000 seconds (~ 2.8 hours). One benefit of using the A-Ω model is that the log-linear plot of strain rates versus strain data clearly shows whether the minimum strain rate has been reached, or not.

For SAC solder at 110°C, the creep rate model is fitted to the data in Figure 14a. The resulting creep curves in Figure 14b indicates that primary creep is significant and that primary and tertiary creep are equally important at about 4500 seconds (75 minutes).

**Application Example # 3: Sn-Cu Creep Data**

The fit of the A-Ω model to Sn0.7Cu creep data at 30°C is shown in Figures 15a and 15b. As in the case of SAC at 110°C, primary creep is significant for an extended period of time. Primary and tertiary creep are equally important at about 7000 seconds (~ 2 hours).

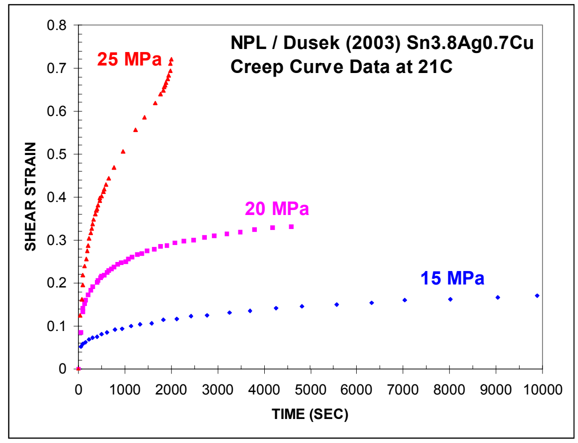
**Stress-Dependence of Parameters A and Ω**

The A-Ω model is fitted to Sn3.5Ag creep data at 25°C and for applied stresses of 18, 24 and 30 MPa, as shown in Figure 16. From Figure 16b, the parameters A and Ω decrease with stress. To a first order, as seen in Figures 17a and 17b, the stress dependence of A and Ω is linear. This dependence is considered weak, compared to the power-law type of dependence for minimum creep rates as a function of stress.

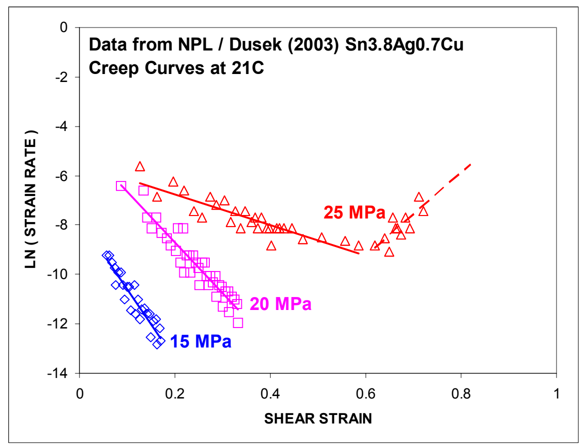
A similar linear and decreasing dependence of Alpha (A) as a function of stress is shown in Figure 18c for SAC solder in shear at 21°C. In this example, the stress-dependence of Ω is not available since, for two of the three stress levels for which creep curves were given (Figure 18a), the test data only shows decelerating strains (Figure 18b).

**Temperature Dependence of Parameters A and Ω**

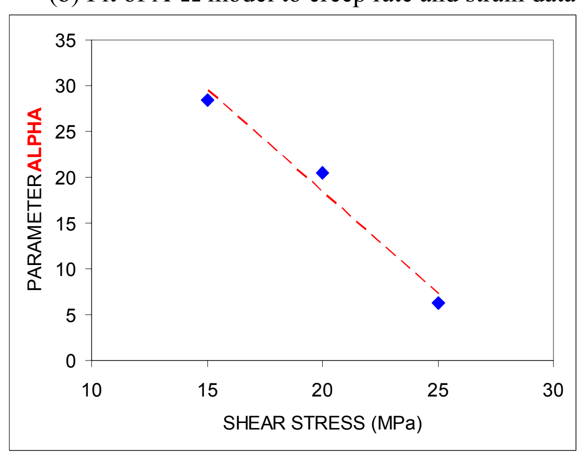
Figure 19 shows the fit of the model to Sn3.5Ag creep rate and creep data at 125°C for an applied stress of 20 MPa. The corresponding values of the A and Ω parameters are as given in Figure 19a: A = 208, Ω = 7.23 at 125C, 20 MPa. Creep data from the same experiment (Kariya et al., 2003) at 25°C was shown earlier in Figure 16 but not at the same stress levels. By interpolation of the stress-dependence of A and Ω in Figure 17, the parameters at 25°C, 20 MPa are estimated as: A = 241 , Ω = 11. From these results at 25°C and 125°C, both A and Ω are seen to decrease with increasing temperature. The temperature dependence is also weak compared to the Arrhenius type of temperature dependence of minimum strain rates. In a separate study of Sn3.9Ag0.6Cu creep data, we found that, to a first order, the temperature dependence of A and Ω was linear over a wide temperature range.




(a) Original creep data



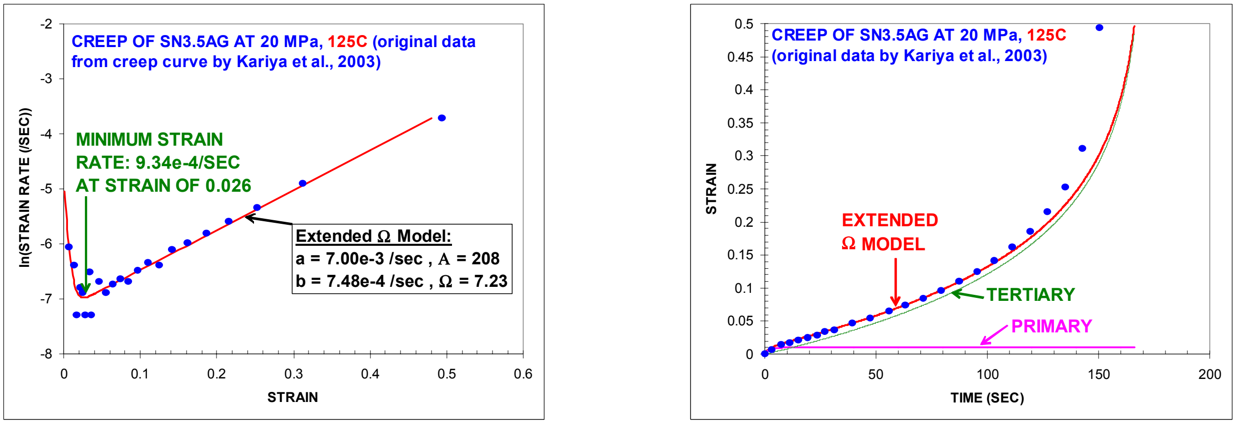

(b) Fit of A-Ω model to creep rate and strain data




(c) Alpha vs. stress

**Figure 18:** Fit of A-Ω model to SAC shear creep data at 21°C (original creep data after Dušek et al., 2003) and stress dependence of parameter Alpha (A).

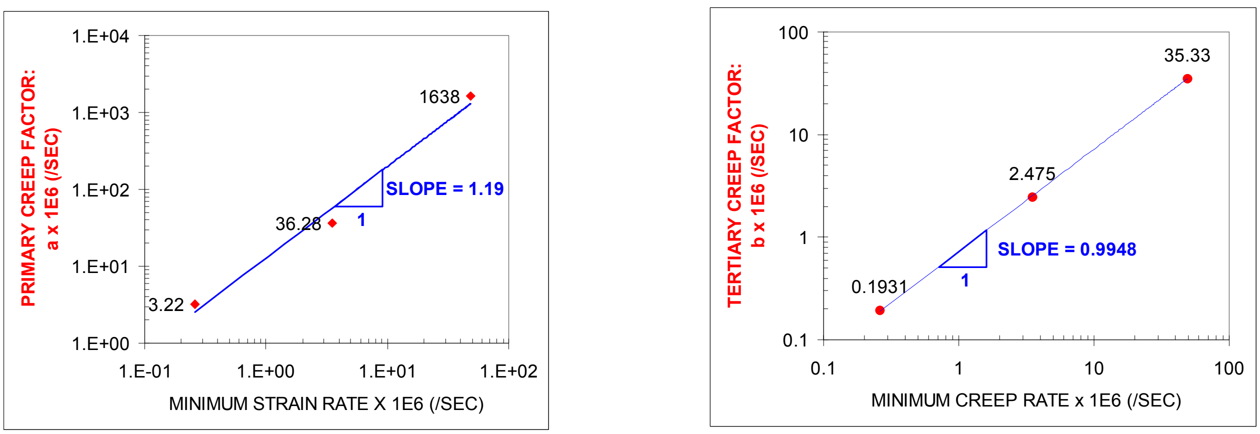
Incidentally, for the creep curve in Figure 19, equation (13) predicts: $t_r \cdot (9.34 \times 10^{-4}) = 1/7.23 \Rightarrow t_r = 148$. The predicted rupture time is 2% less than an estimated rupture time: $t_r$ ~ 150 seconds from the actual creep curve.




(a) Strain rate vs. strain data in log-linear coordinates

(b) Fit of A-Ω model to Sn3.5Ag creep data

**Figure 19:** Fit of A-Ω model to Sn3.5Ag creep rate and creep data at 125°C and an applied stress of 20 MPa (original data after Kariya et al., 2003)




(a) Coefficient "a" vs. minimum strain rate

(b) Coefficient "b" vs. minimum strain rate

**Figure 20:** Creep rate coefficients "a" and "b" vs. minimum strain rate for Sn3.5Ag at 25°C (from fitting of extended W model to strain rate vs. strain data in Figure 16)

**Dependence of Coefficients "a" and "b" on Minimum Strain Rates**

Figure 20 shows plots of the creep rate coefficients "a" and "b" in equation (5) vs. minimum strain rates for Sn3.5Ag at 25°C. The values of the coefficients and of the minimum strain rates are from the fitting of the A-Ω model to the strain rate vs. strain data (Kariya et al., 2003) in Figure 16. From the slopes of trendlines in Figures 20a and 20b, "a" and "b" have an almost linear dependence on minimum creep rates. This suggests that the minimum creep rate can be factored out of equation (5) and the creep rate equation is rewritten as:

$$\overset{\circ}{\varepsilon} = \overset{\circ}{\varepsilon}_{MIN} \cdot \left( a_0 \cdot e^{-A \cdot \varepsilon} + b_0 \cdot e^{\Omega \cdot \varepsilon} \right) \qquad (15)$$

where $a_0$ and $b_0$ are alloy-dependent constants. The entire creep curve is thus strongly dependent on the minimum creep rate. This is also consistent with the application of Monkman-Grant relationships which, to a first order, give creep rupture times that are inversely proportional to minimum creep rates for a variety of Sn-based solders (Plumbridge, 2003; Shohji, et al., 2004). The minimum creep rate itself has a strong stress and temperature dependence (e.g., Clech, 2004).

**Conclusions**

The analysis of creep curves for a variety of Sn-based solders suggests that, while most of them display a minimum creep rate, not much of a steady-state stage is unambiguously evident. Indeed, it appears that creep curves are determined mostly by competing strain hardening, i.e. primary creep with decelerating strains, and weakening or tertiary creep with accelerating strains. This is in agreement with experts in creep of metals who stated:

- "*Unfortunately, the strain vs. time creep curves for most engineering alloys, including solders, bear little resemblance to the textbook depiction*" (Plumbridge, 2003).
- "*Inspection of constant-stress creep curves indicates that a steady-state condition is rarely achieved*" (Wilshire, 2002).
- "*With most metals and alloys, a minimum rather than a steady creep rate occurs*" (Wilshire, 2002).

The latter observations led Wilshire (2002) to abandon classical creep models and develop an alternative model for engineering metals and alloys, the Theta projection concept, which is a time-dependent model that describes entire creep curves. A modified Theta projection concept was also developed by Kariya et al. (2003) to describe creep curves for Sn3.5Ag solder.

The approach followed in this paper is entirely different since one essential requirement of developing a constitutive model for simulation purposes is that the model should satisfy the principle of objectivity, whereby strain rates are invariant under a change of time referential (e.g., time translation). The proposed A-Ω model satisfies this requirement, and more specifically, strain rates decay exponentially with strains in the primary creep region and increase exponentially with strains in the tertiary creep region.

The A-Ω model is an extension of the Omega method (Prager, 2000) for tertiary creep of steels to primary and tertiary creep of Sn-based solders. For small strains in the primary creep region, the A-Ω model reduces to the Pao-Marin (1957) kinetic type of primary creep model. In the tertiary creep region, the model obviously reduces to the Omega method. As in the Omega method, this leads to a Monkman-Grant type of relationship for creep rupture times that are equal to the inverse of Ω times the minimum creep rate. This allows for quick estimates of creep rupture times without carrying a creep test to rupture and without an empirical correlation of creep rupture data.

Other results of this study and properties of the A-Ω model are:

- The A-Ω creep model handles both hardening and softening of Sn-based solders and predicts the entire creep curve. A is a strain-hardening rate parameter, Ω is a creep damage or recovery rate parameter.
- Transition from hardening to softening is dictated by strains at the minimum creep rate, not time. This is at a departure from classical, time-based models for creep of solders where the transition is assumed to occur following an extended stage of steady-state creep.
- Non-linear curve fitting of the A-Ω model to creep data on a log-linear plot of strain rate vs. strain is straightforward with no apparent convergence problems.
- As in the original Ω method for tertiary creep of steels, the main parameters A and Ω can be determined without conducting creep tests to rupture. This allows for a significant reduction in test time. By tracking ln(strain rate) vs. strain data as a creep test is in progress, users can clearly tell when the minimum rate is reached, if it is, and when there is enough data to get good estimates of A and/or Ω.
- The parameters A and Ω have a weak dependence on, and decrease linearly with increasing stress and temperature.
- As for most material properties, values of the creep curve parameters A and Ω for a given set of stress and temperature conditions are sensitive to the solder alloy composition, casting or soldering conditions, cooling rate, ageing treatment etc…
- The coefficients "a" and "b" of the creep rate model have a linear dependence on the minimum creep rate.
- The applicability of the model to a variety of Sn-based solders, including Sn3.5Ag, Sn0.7Cu and SAC alloys, was demonstrated through the analysis of numerous creep curves.

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